MATH 1650: QUADRATIC FUNCTIONS

EXAMPLE: Let $f(x) = 2(x-1)^2 + 3$.

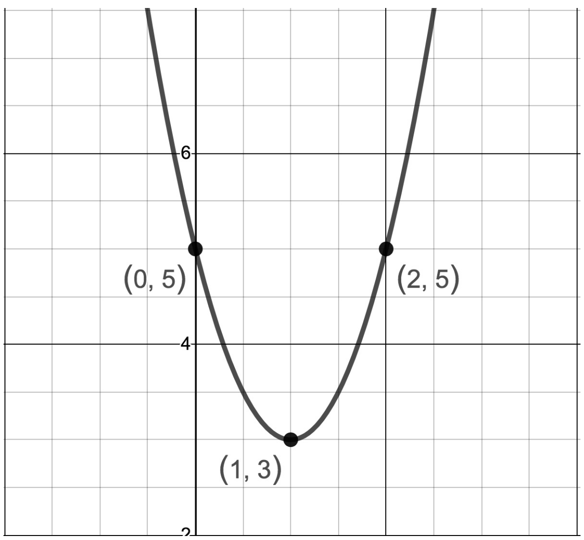
- Vertex: $(1, 3)$.

- $f(0) = 2((0)-1)^2 + 3 = 5$ so the y-intercept is $(0, 5)$.

- Setting $f(x) = 2(x-1)^2 + 3 = 0$ gives $2(x-1)^2 = -3$, or $(x-1)^2 = -\frac{3}{2}$.

  Since $(x-1)^2$ is always positive or 0, there are no solutions.

- Graph $y = f(x)$ using the vertex, y-intercept, and symmetry.



the **domain** of f: $(-\infty, \infty)$

the **range** of f: $[3, \infty)$

the **maximum** of f: none

the **minimum** of f: 3

f is:

increasing: $[1, \infty)$

decreasing: $(-\infty, 1]$

constant: nowhere

EXAMPLE: Let $f(x) = -\frac{1}{2}(x-1)^2 + 8$.

- Vertex: $(1, 8)$.

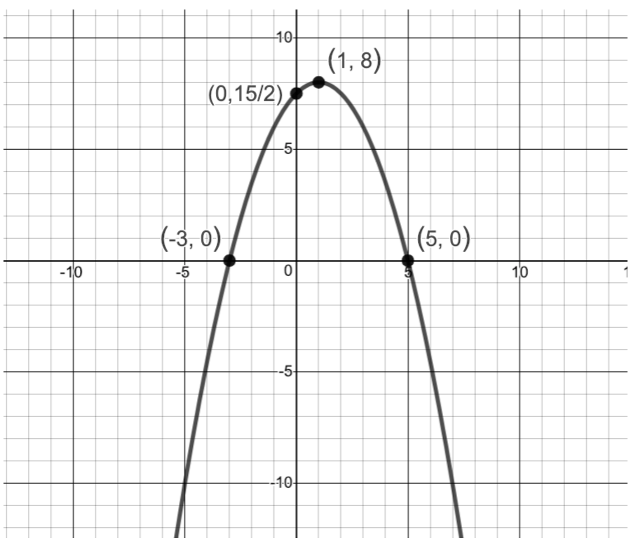
- Find $f(0) = -\frac{1}{2}((0)-1)^2 + 8 = \dfrac{15}{2}$ so the y-intercept is $\left(0, \dfrac{15}{2}\right)$.

- Solve $f(x) = 0$:

$$\begin{aligned} -\tfrac{1}{2}(x-1)^2 + 8 &= 0 \\ -\tfrac{1}{2}(x-1)^2 &= -8 \\ (x-1)^2 &= 16 \\ x - 1 &= \pm\sqrt{16} = \pm 4 \end{aligned}$$

  From $x - 1 = 4$, we get $x = 5$; from $x - 1 = -4$, we get $x = -3$. The x-intercepts are $(-3, 0)$ and $(5, 0)$.

- Graph $y = f(x)$ using the vertex, y-intercept, x-intercepts, and symmetry.



the **domain** of f: $(-\infty, \infty)$

the **range** of f: $(-\infty, 8]$

the **maximum** of f: 8

the **minimum** of f: none

f is:

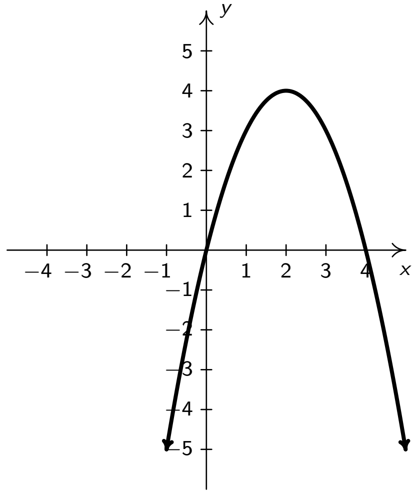
increasing: $(-\infty, 1]$

decreasing: $[1, \infty)$

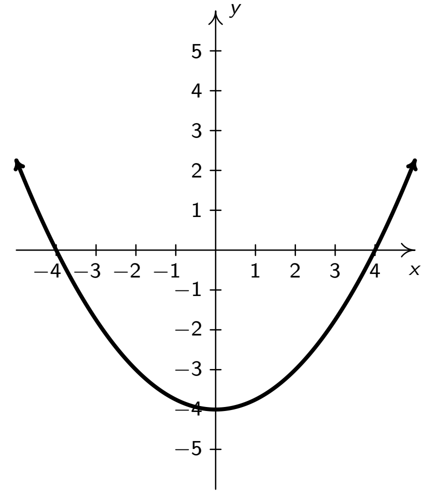
constant: nowhere

EXAMPLE: Find the **standard form** of the quadratic functions whose graphs are shown below:

- The graph of $y = f(x) = -(x-2)^2 + 4$:



- The graph of $y = g(x) = \dfrac{1}{4}x^2 - 4$



EXAMPLE: Let $f(x) = 6x - x^2$.

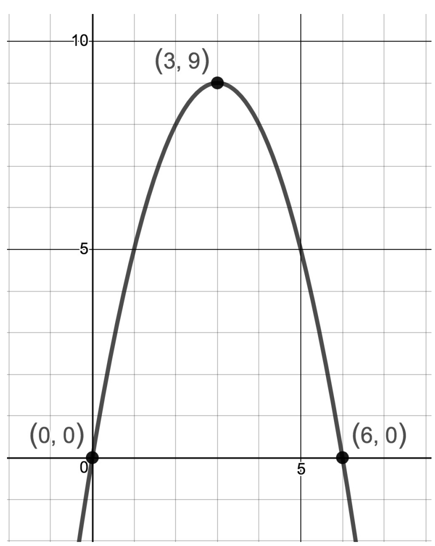
- Vertex: $x = -\dfrac{b}{2a} = -\dfrac{6}{2(-1)} = 3$. $y = f(3) = 6(3) - (3)^2 = 9$. So the vertex is $(3, 9)$.

- $f(0) = 6(0) - (0)^2 = 0$. The y-intercept is $(0, 0)$.

- Solving $f(x) = 0$: to solve $6x - x^2 = 0$, we factor: $x(6 - x) = 0$ so $x = 0$ or $x = 6$.

  The x-intercepts are $(0, 0)$ and $(6, 0)$.

- Graph $y = f(x)$ using the vertex, y-intercept, x-intercepts, and symmetry.



the **domain** of f: $(-\infty, \infty)$

the **range** of f: $(-\infty, 9]$

the **maximum** of f: 9

the **minimum** of f: none

f is:

increasing: $(-\infty, 3]$

decreasing: $[3, \infty)$

constant: nowhere

EXAMPLE: Let $f(x) = x^2 - 3x - 4$.

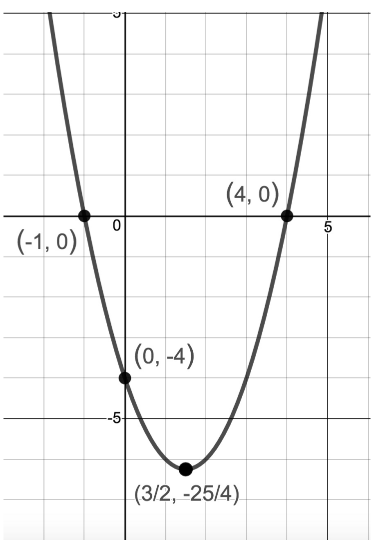
- Vertex: $x = -\dfrac{b}{2a} = -\dfrac{-3}{2(1)} = \dfrac{3}{2}$. $y = f\left(\dfrac{3}{2}\right) = \left(\dfrac{3}{2}\right)^2 - 3\left(\dfrac{3}{2}\right) - 4 = -\dfrac{25}{4}$. So the vertex is $\left(\dfrac{3}{2}, -\dfrac{25}{4}\right)$.

- $f(0) = (0)^2 - 3(0) - 4 = -4$ so the y-intercept is $(0, 4)$.

- Solving $f(x) = 0$: to solve $x^2 - 3x - 4 = 0$, we factor: $(x-4)(x+1) = 0$. We get $x = 4$ or $x = -1$.

  The x-intercepts are $(-1, 0)$ and $(4, 0)$.

- Graph $y = f(x)$ using the vertex, y-intercept, x-intercepts, and symmetry.



the **domain** of f: $(-\infty, \infty)$

the **range** of f: $\left[-\dfrac{25}{4}, \infty\right)$

the **maximum** of f: none

the **minimum** of f: $-\dfrac{25}{4}$

f is:

increasing: $\left[\dfrac{3}{2}, \infty\right)$

decreasing: $\left(-\infty, \dfrac{3}{2}\right]$

constant: nowhere

EXAMPLE: Let $f(x) = 6 - 2x - x^2$.

- Vertex: $x = -\dfrac{b}{2a} = -\dfrac{-2}{2(-1)} = -1$. $y = f(-1) = 6 - 2(-1) - (-1)^2 = 7$. So the vertex is $(-1, 7)$.

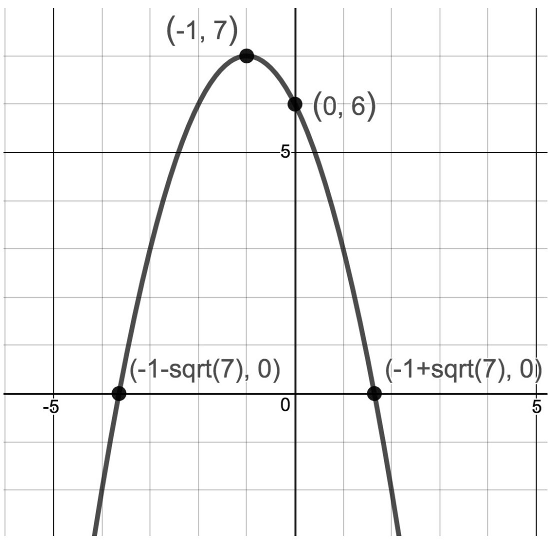
- Find $f(0) = 6 - 2(0) - (0)^2 = 6$ so the y-intercept is $(0, 6)$.

- Solving $f(x) = 0$: to solve $6 - 2x - x^2 = 0$ we rewrite as $x^2 + 2x - 6 = 0$ and use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-6)}}{2(1)} = \frac{-2 \pm \sqrt{28}}{2} = \frac{-2 \pm 2\sqrt{7}}{2} = \frac{2\left(-1 \pm \sqrt{7}\right)}{2} = -1 \pm \sqrt{7}.$$

  The x-intercepts are: $(-1 - \sqrt{7}, 0)$ and $(-1 + \sqrt{7}, 0)$.

- Graph $y = f(x)$ using the vertex, y-intercept, x-intercepts, and symmetry.



the **domain** of f: $(-\infty, \infty)$

the **range** of f: $(-\infty, 7]$

the **maximum** of f: 7

the **minimum** of f: none

f is:

increasing: $(-\infty, -1]$

decreasing: $[-1, \infty)$

constant: nowhere

MATH 1650: APPLICATIONS OF QUADRATIC FUNCTIONS

EXAMPLE: Assume we make exactly the same amount of T-shirts we sell, 'x.'

- $R(x) = x(30 - 2x) = 30x - 2x^2$. Since $p(x)$ is valid only for $0 \le x \le 15$, so is $R(x)$:

$$R(x) = -2x^2 + 30x, \quad 0 \le x \le 15.$$

- $P(x) = R(x) - C(x) = \left(30x - 2x^2\right) - (2x + 26) = -2x^2 + 28x - 26$.

  $C(x)$ is valid only for $x \ge 0$ and $R(x)$ is valid only for $0 \le x \le 15$, so $P(x)$ is valid only for $0 \le x \le 15$.

$$P(x) = -2x^2 + 28x - 26, \quad 0 \le x \le 15.$$

- $P(0) = -2(0)^2 + 28(0) - 26 = -26$ which means if we sell 0 shirts, we are in debt \$26.

  NOTE: It is no coincidence that our start-up costs were \$26!

- Solving $P(x) = 0$: to solve $-2x^2 + 28x - 26 = 0$, we factor:

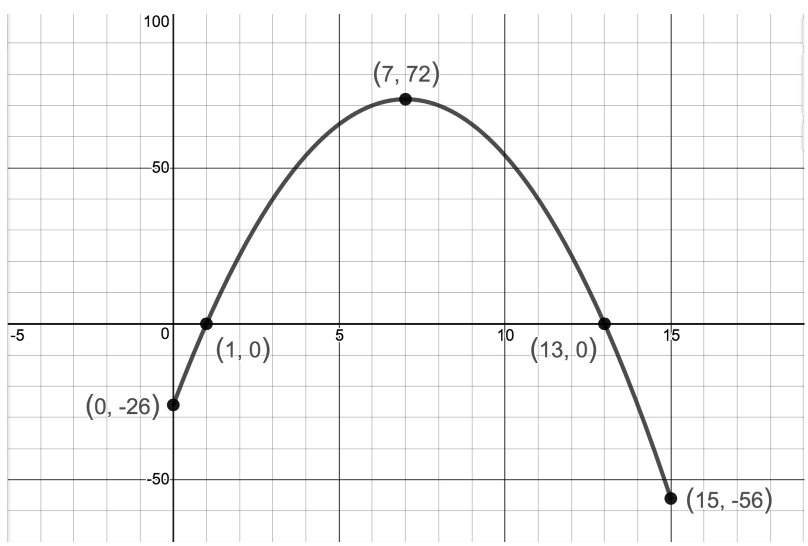
$$-2x^2 + 28x - 26 = -2\left(x^2 - 14x + 13\right) = -2(x-1)(x-13).$$

  Solving $-2(x-1)(x-13) = 0$, we get $x = 1$ and $x = 13$. Hence, when making 1 or 13 shirts, profit is \$0. In other words, we 'break even' at these points – our costs perfectly balance our revenue.

- Vertex: $x = -\dfrac{b}{2a} = -\dfrac{28}{2(-2)} = 7$, $y = P(7) = -2(7)^2 + 28(7) - 26 = 72$. The vertex is at $(7, 72)$.

  This means if we make and sell 7 shirts, we will earn \$72 in profit.

  Moreover, because the graph opens downwards, we know \$72 is the maximum profit obtainable.



  NOTE: The domain of P is restricted to $0 \le x \le 15$, so our graph starts at $(0, -26)$ and ends at $(15, -56)$.

- $ARC_{[7,14]} = \dfrac{P(14) - P(7)}{14 - 7} = \dfrac{-98}{7} = -14 = \dfrac{-\$14\,\text{profit}}{1\,\text{shirt}}$.

  When making and selling between 7 and 14 shirts, we are losing an average of \$14 in profit per shirt.